Rossmoyne Senior High School

Semester One Examination, 2019

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 3

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student number: In figures | | | | | | | | |

In words: ______________________________

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Circle your teacher's name: Ms Chua Ms Robinson Mr Tan

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	50	35
Section Two: Calculator-assumed	13	13	100	99	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 **(6 marks)**

Consider the following system of equations, where a and b are constants.

$$\begin{aligned} x + 2y + 2z &= 5 \\ 3x + 2y + 4z &= 9 \\ 3x + ay + 2z &= b \end{aligned}$$

For each of the following cases, determine the number of solutions that exist for the system and give a geometric interpretation of the situation.

(a) $a = 1,\ b = 3.$ (3 marks)

(b) $a = -2,\ b = 3.$ (3 marks)

Question 9 **(7 marks)**

(a) Determine the values of the real constant a and the real constant b given that $z-4+2i$ is a factor of z^3+az+b.

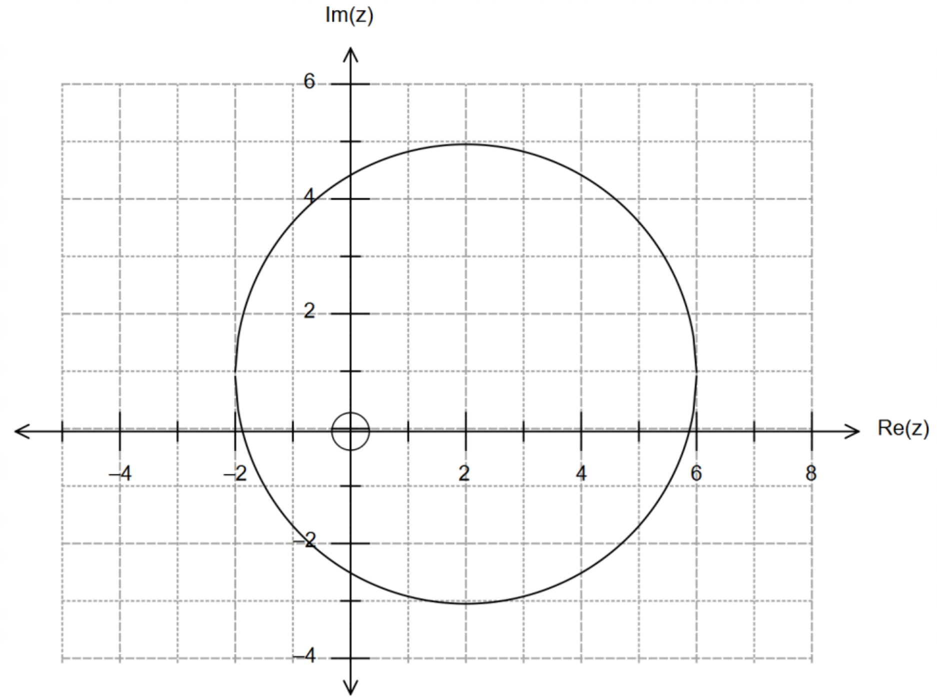
(4 marks)

(b) (i) Clearly show that $2+i$ is a root of the equation $z^3-7z^2+17z-15=0$. (2 marks)

(ii) State all three solutions of $z^3-7z^2+17z-15=0$. (1 mark)

Question 10 **(6 marks)**

The diagram below shows the region represented by $|z - 2 - i| = 4$



(a) Determine the minimum and maximum value of $|z|$. (2 marks)

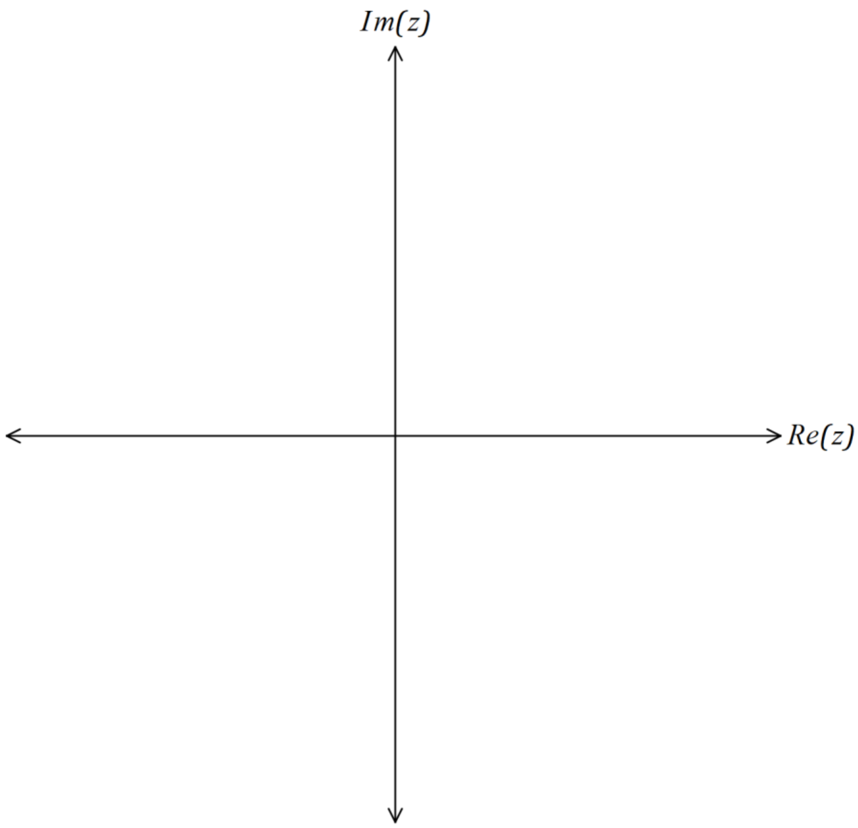
(b) Determine the value(s) of $arg(z)$ for when $Re(z) = 4$. (4 marks)

Question 11 **(9 marks)**

Let $w = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$.

(a) Express w, w^2, w^3 and w^4 in the form $r\,\text{cis}\,\theta, -\pi < \theta \leq \pi$. (2 marks)

(b) Sketch w, w^2, w^3 and w^4 as vectors on the Argand diagram below. (2 marks)



(c) Describe the transformation in the complex plane of any point z when it is multiplied by w. (2 marks)

(d) Simplify

(i) $w + w^3 + w^5 + w^7$. (1 mark)

(ii) $w + w^3 + w^5 + \cdots + w^{2017} + w^{2019}$. (2 marks)

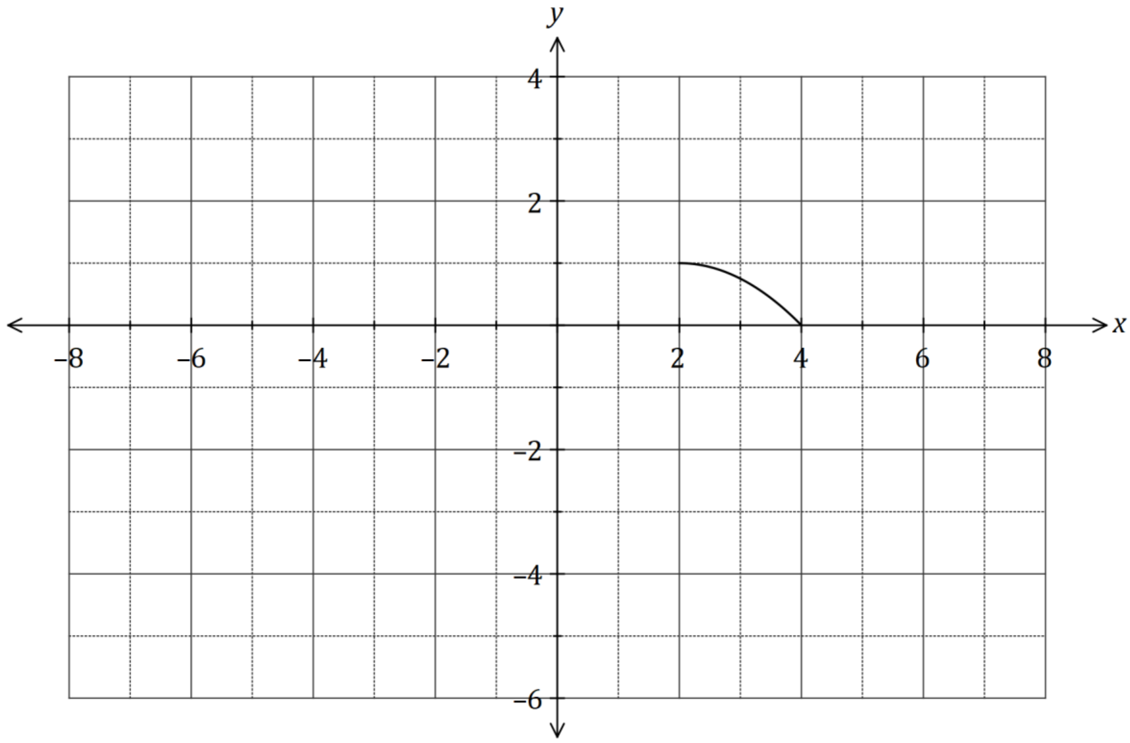
Question 12 **(9 marks)**

The position vector of a small body, in metres, is $\mathbf{r}(t) = (2 + 4\sin(t))\mathbf{i} + (2\cos(2t) - 1)\mathbf{j}$ where t is the time in seconds since motion began.

(a) Show that the body is stationary when $t = \frac{\pi}{2}$ and state its position at this time. (3 marks)

(b) Derive the Cartesian equation of the path of the body. (4 marks)

(c) Complete the following plot to show the path taken by the body. (2 marks)



Question 13 **(7 marks)**

(a) Solve the equation $z^5 + 32 = 0$, writing your solutions in polar form $r\,\text{cis}\,\theta$. (4 marks)

(b) Use your answers from (a) to show that $\cos\left(\frac{\pi}{5}\right) + \cos\left(\frac{3\pi}{5}\right) = \frac{1}{2}$. (3 marks)

Question 14 **(8 marks)**

The position vectors of two particles at time t are given below, where a is a constant.

$$\mathbf{r}_{\mathrm{A}} = 21\mathbf{i} - 11\mathbf{j} + 14\mathbf{k} + t(-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \text{ and } \mathbf{r}_{\mathrm{B}} = 5\mathbf{i} - 3\mathbf{j} + a\mathbf{k} + t(5\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

The paths of the particles cross at P but the particles do not meet.

(a) Determine the value of the constant a and the position vector of P. (5 marks)

(b) Show that the point $(4, 10, 1)$ lies in the plane containing the two lines. (3 marks)

Question 15 **(8 marks)**

Sphere S has diameter PQ, where P and Q have coordinates $(2, -3, 1)$ and $(-4, 7, 5)$ respectively.

(a) Determine the vector equation of the sphere. (3 marks)

(b) Show that the point $(1, -1, 2)$ lies inside the sphere. (2 marks)

(c) Show that the line with equation $\mathbf{r} = \begin{pmatrix} 1 \\ -3 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is tangential to the sphere.

(3 marks)

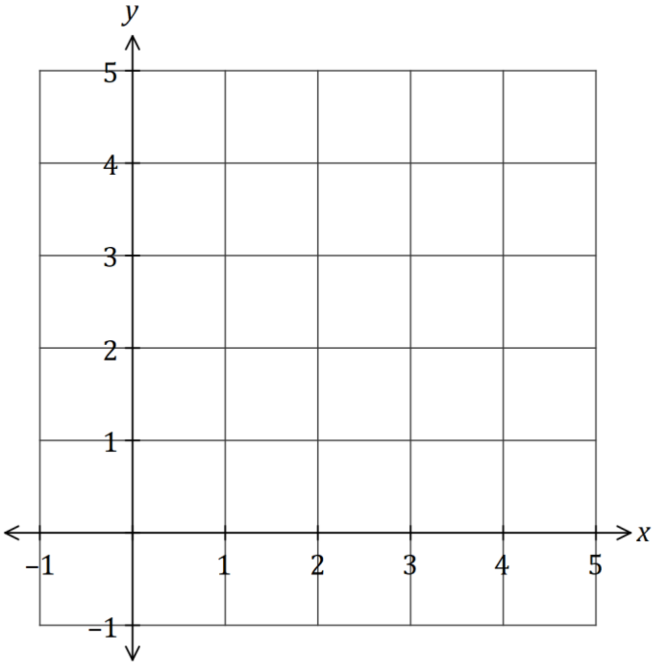
Question 16 **(9 marks)**

Let $f(x) = \sqrt{x-1}$, $g(x) = \dfrac{3}{x}$ and $h(x) = f \circ g(x)$.

(a) Determine an expression for $h(x)$ and show that the domain of $h(x)$ is $0 < x \leq 3$. (3 marks)

(b) Determine an expression for $h^{-1}(x)$, the inverse of $h(x)$. (1 mark)

(c) Sketch the graphs of $y = h(x)$ and $y = h^{-1}(x)$ on the axes below. (3 marks)



(d) Solve $h(x) = h^{-1}(x)$, correct to 0.01 where necessary. (2 mark)

Question 17 **(9 marks)**

A pole and a wall stand vertically on horizontal ground. A small projectile is launched from the pole at a height of 3.16 m above the ground and sometime later hits the wall at a height of 1.79 m above the ground. The projectile has an initial velocity of 32 ms^{-1} at an angle of $36°$ above the horizontal.

Any effects of air resistance and wind can be ignored. Let $\mathbf{i}$ and $\mathbf{j}$ be unit vectors in the horizontal and vertical (upward) directions and the foot of the pole be at $(0, 0)$.

The acceleration acting on the projectile is given by $\mathbf{a}(t) = -9.8\mathbf{j}$ ms^{-2}.

(a) Use the information above to derive vector equations for the velocity $\mathbf{v}(t)$ and displacement $\mathbf{r}(t)$ of the projectile at any time t. (3 marks)

(b) Determine

(i) the time that the projectile takes to travel between the pole and the wall. (2 marks)

(ii) the speed of the projectile at the instant it hits the wall. (2 marks)

(iii) the distance travelled by the projectile between the pole and the wall. (2 marks)

Question 18 **(7 marks)**

(a) Point A has coordinates $(-6, 1, 4)$ and plane Π has equation $2x + y - 2z = 17$. Determine

(i) a vector equation for the straight line through A perpendicular to Π.

(1 mark)

(ii) the perpendicular distance of A from Π. (3 marks)

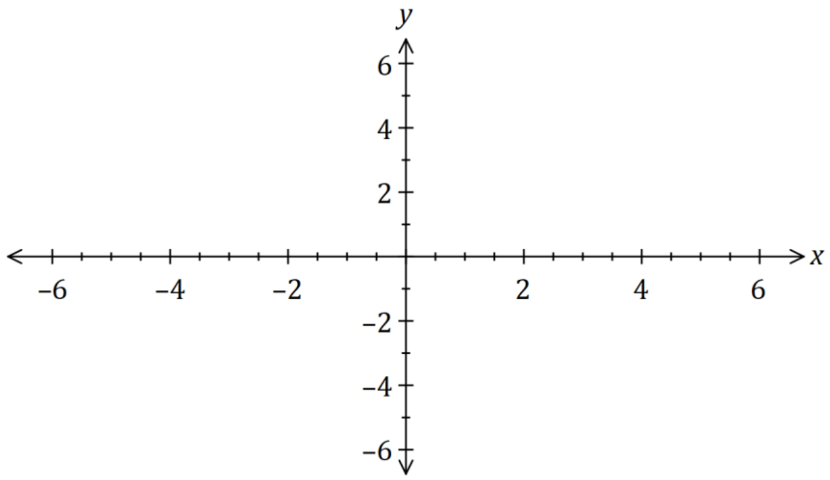
(b) Prove that the perpendicular distance from the origin to the plane $\mathbf{r} \cdot \mathbf{n} = k$ is $\dfrac{k}{|\mathbf{n}|}$.

(3 marks)

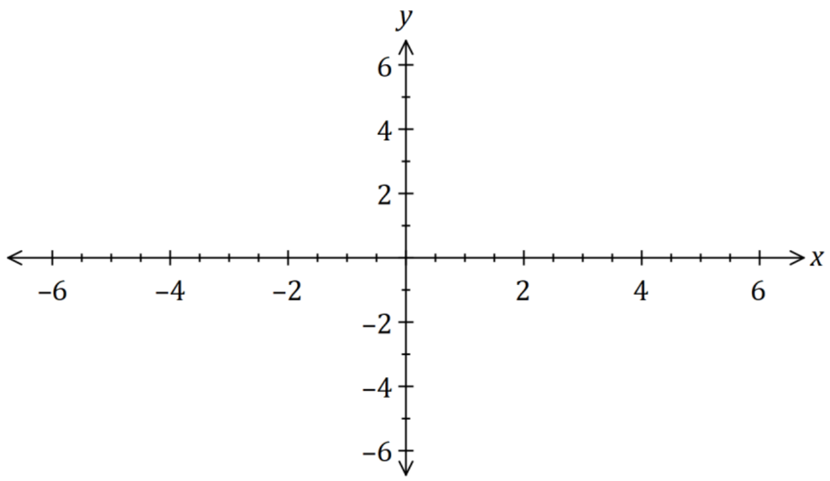
Question 19 **(8 marks)**

Let $f(x) = 6 - |3x - 6|$.

(a) Sketch the graph of $y = f(x)$ on the axes below. (2 marks)



(b) Sketch the graph of $y = f(|x|)$ and hence solve $f(|x|) - 3 = 0$. (3 marks)

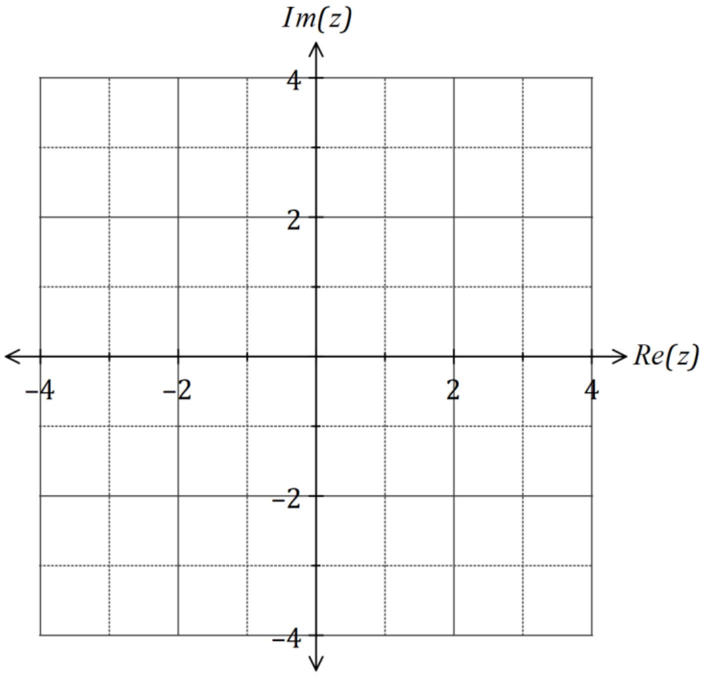


(c) Let $g(x) = a|x + b| + c$. The equation $f(x) = g(x)$ is true only for $-1 \leq x \leq 2$. Determine the value of each of the constants a, b and c. (3 marks)

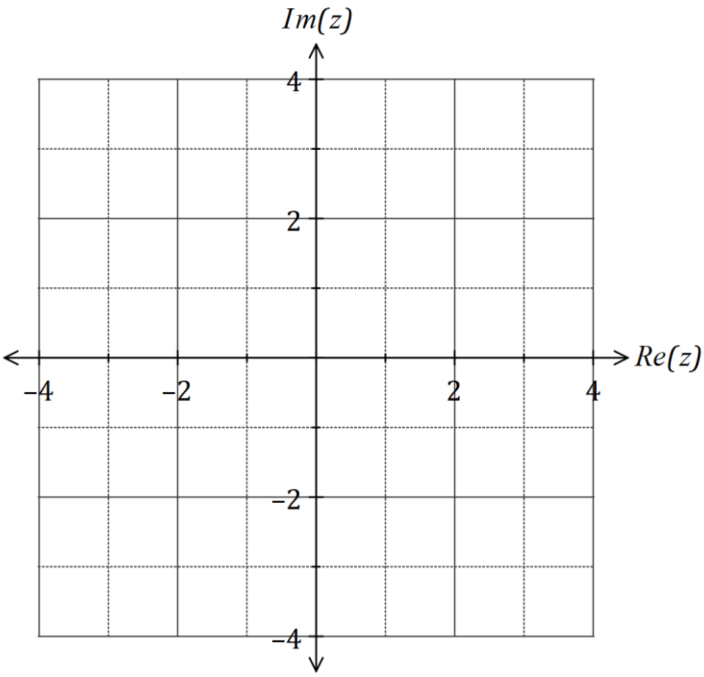
Question 20 **(6 marks)**

Sketch the locus of the complex number z given by

(a) $|z+1+i| \leq |z-1-3i|$. (3 marks)



(b) $|z+3| = |z|+3$. (3 marks)



End of questions

Supplementary page

Question number: _________